UNIVERSITY OF CAMBRIDGE

Economics Admissions Assessment

Specimen Paper Section 1: Explained Answers

New Format for 2020

MATHEMATICS

1

The answer is option **A**.

We can write:

area of shape $=$ area of square $-$ area of semi-circle.

If we let the length of one side of the square be $2x$, so that the radius of the semi-circle is x, then this becomes:

$$100 = 4x^2 - \frac{1}{2}\pi x^2 = x^2\left(4 - \frac{\pi}{2}\right)$$

Rearranging:

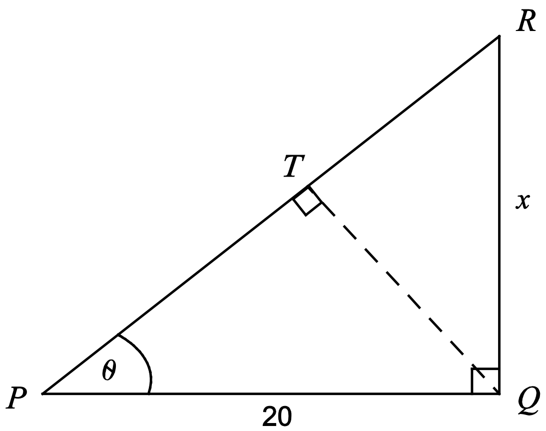
$$x = \sqrt{\frac{200}{8-\pi}} = 10\sqrt{\frac{2}{8-\pi}}$$

So the length of one side of the square is $2x = 20\sqrt{\frac{2}{8-\pi}}$

2

The answer is option **E**.

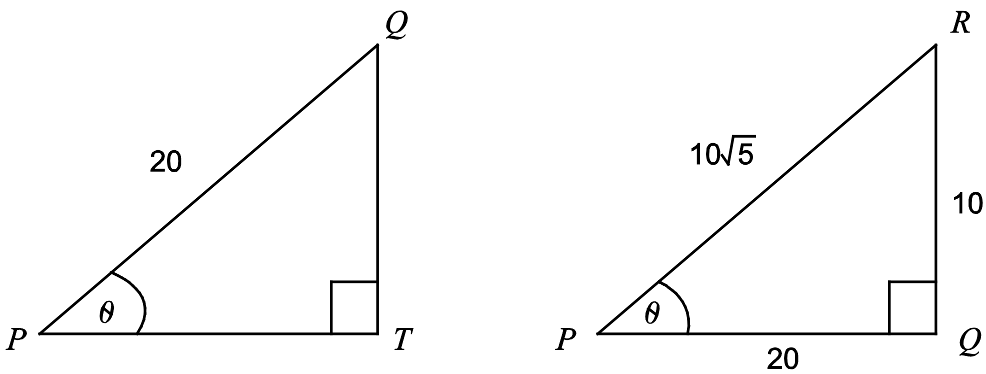
Let $RQ = x$



We know $RQ : PQ$ is $1 : 2$ so we can deduce $x = 10$

By Pythagoras' Theorem, we have $PR = 10\sqrt{5}$

We notice triangles PQT and PRQ are similar:

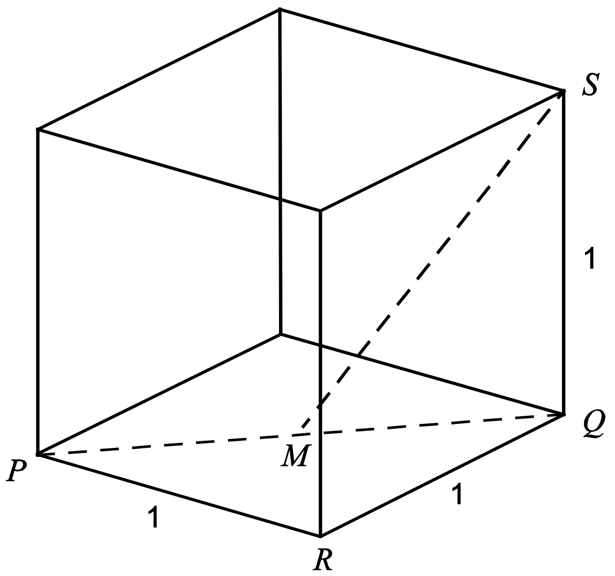


So we can write: $\frac{QT}{10} = \frac{20}{10\sqrt{5}}$

Giving: $QT = \frac{20}{\sqrt{5}} = 4\sqrt{5}$

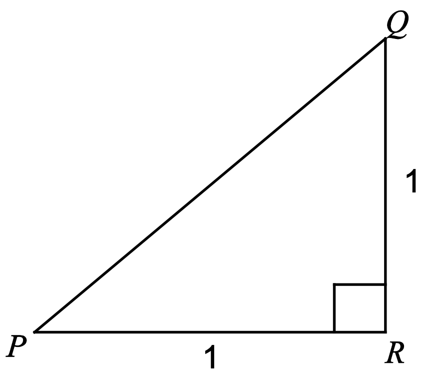
3

The answer is option **A**.



[diagram not to scale]

In triangle PQR, the angle at R is a right angle so we can use Pythagoras' Theorem to find PQ:

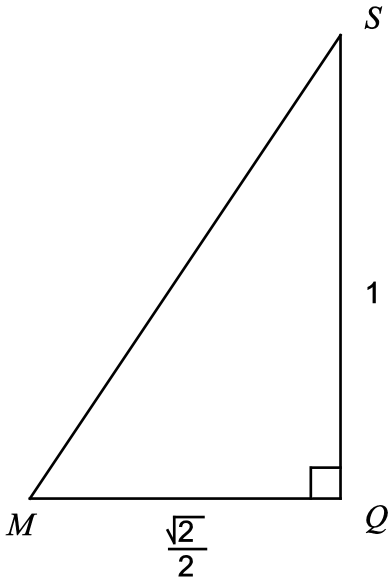


$PQ^2 = 1^2 + 1^2 = 2$

Giving $PQ = \sqrt{2}$

QM is half of PQ so $QM = \dfrac{\sqrt{2}}{2}$

Now looking at triangle MQS:



The angle at Q is a right angle so we can use Pythagoras' Theorem:

$$MS^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + 1^2 = \frac{2}{4} + 1 = \frac{3}{2}$$

Giving $MS = \sqrt{\frac{3}{2}}$

4

The answer is option **D**.

When the minute hand is pointing to the 9 [indicating 45 minutes past the hour], the hour hand will have moved three-quarters of the angle between the 9 and the 10 on the clock dial. The angle between the 9 and the 10 is 30 degrees so the hour hand will have moved three-quarters of 30 degrees, or 22.5 degrees. The angle between the hands is, therefore, 22.5°.

5

The answer is option **B**.

To answer this we shall use:
area of triangle $= \frac{1}{2} \times$ base $\times$ height
This gives:

$$\text{area } = \frac{1}{2}\left(4+\sqrt{2}\right)\left(2-\sqrt{2}\right) = \frac{1}{2}\left(8-4\sqrt{2}+2\sqrt{2}-\sqrt{2}\sqrt{2}\right) = \frac{1}{2}\left(6-2\sqrt{2}\right) = 3-\sqrt{2}$$

6

The answer is option **D**.

As the cylinder has the same internal diameter as the sphere, it has the same radius, r. As the length of the cylinder is the same as the diameter of the sphere, it has length $2r$. So we can write:

Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of cylinder $=$ base area $\times$ height $= \pi r^2 \times 2r = 2\pi r^3$

This then allows us to calculate the fraction of the space in the cylinder taken up by the sphere:

$$\frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{2}{3}$$

7

The answer is option **A**.

For $0 < x < 1$, we have:

$$0 < x^2 < 1$$

$$0 < \sqrt{x} < 1$$

and also:

$$1 < \frac{1}{x}$$

$$\frac{1}{\sqrt{x}} < \frac{1}{x}$$

$$\frac{1}{1+x} < \frac{1}{x}$$

so $\frac{1}{x}$ has the largest value in $0 < x < 1$

8

The answer is option **C**.

Using the fact that triangle ABC is similar to triangle ADE [they have the same angles], we can write:

$$\frac{AB}{AD} = \frac{BC}{DE}$$

This gives:

$$\frac{4}{x} = \frac{x}{x+3}$$

From which we get:

$$4(x+3) = x^2$$

Rearranging:

$$x^2 - 4x - 12 = 0$$

Factorising:

$$(x-6)(x+2) = 0$$

Giving $x = 6$ [as we cannot have a negative value for x as it represents a length].

And so $DE = x + 3 = 6 + 3 = 9$

9

The answer is option **C**.

$P \propto \dfrac{1}{Q^2}$ so $P = \dfrac{k}{Q^2}$ for some k

We can use multiplying factors to work out how each symbol changes; for instance, as Q increases by 40% its new value is $1.4Q$

For this question, we have:

$Q_{new} = 1.4Q$

Giving:

$$P_{new} = \frac{k}{(1.4Q)^2} = \frac{k}{1.96Q^2} = \frac{1}{1.96}P$$

We then notice:

$\dfrac{1}{1.96} \approx \dfrac{1}{2}$ and $\dfrac{1}{1.96} > \dfrac{1}{2}$

so P_{new} is a little over 50% of P and so we can deduce that P has decreased by a little under 50%.

10

The answer is option **D**.

We are given: $x \propto z^2$ and $y \propto \dfrac{1}{z^3}$ which allows us to write: $x = kz^2$ and $y = \dfrac{c}{z^3}$

We want to eliminate z so we need both expressions to have the same power of z in them:

$x^3 = k^3 z^6$ and $y^2 = \dfrac{c^2}{z^6}$

Eliminating z and ignoring the constants gives:

$$x^3 \propto \frac{1}{y^2}$$

11

The answer is option **C**.

To make the question easier to answer, we can let $QX = 2$

We then find, using the given ratios, that $PX = 12$ and $XR = 3$ and so $PR = 15$

As M is the midpoint of PR then $PM = 7.5$ and so $MX = 4.5$

We can then calculate the ratio asked for:

$$\frac{QX}{MX} = \frac{2}{4.5} = \frac{4}{9}$$

12

The answer is option **D**.

$x^2 \geq 8 - 2x$

Rearranging:

$x^2 + 2x - 8 \geq 0$

Factorising:

$(x+4)(x-2) \geq 0$

Giving:

$x \geq 2$ or $x \leq -4$

13

The answer is option **A**.

We are told:

surface area of cylinder $=$ volume of cylinder

We can write this as follows:

$2\pi r^2 + 2\pi rh = \pi r^2 h$

Rearranging:

$2r = rh - 2h$

$2r = h(r-2)$

Giving:

$\frac{2r}{r-2} = h$

14

The answer is option **D**.

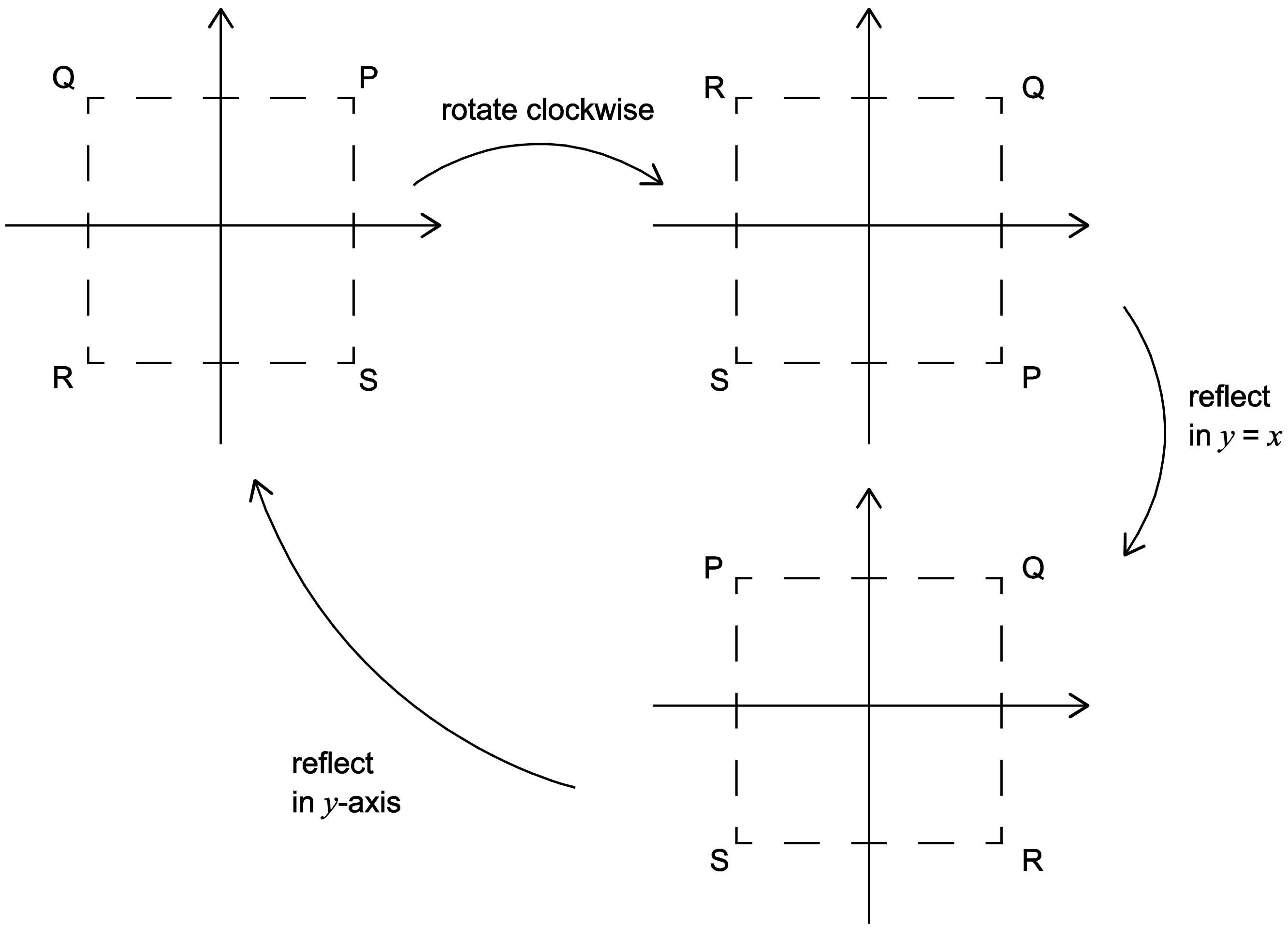
We require: $6 < 2\sqrt{n} < 8$

Which gives, on squaring: $36 < 4n < 64$ or $9 < n < 16$

This is satisfied when $n = 10,11,12,13,14,15$, i.e. there are 6 integers.

15

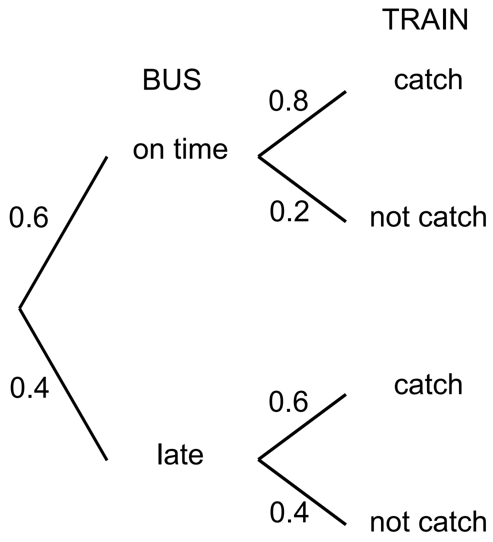
The answer is option **B**.



16

The answer is option **E**.

A tree diagram for this question is:



As Sylvie catches the train we have two possible situations: either the bus was on time, or the bus was late.

Probability that Sylvie catches the train

$=$ probability bus on time and catches train $+$ probability bus late and catches train

$= (0.6 \times 0.8) + (0.4 \times 0.6) = 0.48 + 0.24 = 0.72$

Probability that the bus was on time and Sylvie catches the train $= (0.6 \times 0.8) = 0.48$

So the required probability is: $\frac{0.48}{0.72} = \frac{48}{72} = \frac{2}{3}$

17

The answer is option **C**.

We can let the length of a side of the large square be 3 to help us find the area scale factor.

First we use Pythagoras' theorem to find the length of one side of the first inscribed square:

Length of side of inscribed square $= \sqrt{1^2 + 2^2} = \sqrt{5}$

This allows us to work out the area scale factor using:

Area of main square $=$ 9

Area of inscribed square $=$ 5

Area scale factor $= \dfrac{5}{9}$

Therefore the area of the fourth square [the third inscribed square] as a fraction of the area of the first square must be:

$$\left(\frac{5}{9}\right)^3 = \frac{125}{729}$$

18

The answer is option **C**.

We can use multiplying factors to work out how each symbol changes; for instance, as x increases by 50% its new value is $1.5x$

After the percentage changes the expression becomes:

$$\frac{(1.5x + 1.5y)^2\, 0.8z}{2P} Q$$

This simplifies:

$$\frac{1.5^2\,(x+y)^2\, 0.8z}{2P} Q = \frac{1.5^2 \times 0.8}{2} \times \frac{(x+y)^2 z}{P} Q = 0.9\,\frac{(x+y)^2 z}{P} Q$$

giving a 10% decrease in the value of M.

19

The answer is option **B**.

Prob of scoring $12 =$ (Prob of scoring 6 on fair dice) $\times$ (Prob of scoring 6 on biased dice)

As the question tells us that the Prob of scoring $12 = \frac{1}{18}$

We can write: $\frac{1}{6} \times$ (Prob of scoring 6 on biased dice) $= \frac{1}{18}$

So: (Prob of scoring 6 on biased dice) $= \frac{1}{3}$

As the probability of scoring a 1 or 2 or 3 or 4 or 5 is the same on the biased dice, we can deduce that:

(Prob of scoring 1 on the biased dice) $= \frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$

And so this gives:

Prob of scoring 2 $=$ (Prob of scoring 1 on fair dice) $\times$ (Prob of scoring 1 on biased dice)

or

Prob of scoring 2 $= \frac{1}{6} \times \frac{2}{15} = \frac{1}{45}$

20

The answer is option **H**.

A regular pentagon has 5 sides, so each exterior angle is: $\frac{360}{5} = 72°$

At each vertex of the pentagon the course increases its bearing by 72°.

By the third leg there have been 2 turns so we need to subtract $2 \times 72 = 144°$ from the bearing to get the first leg bearing: $110 - 144 = -34°$

So first leg bearing is 34° beyond North in an anti-clockwise direction.

The bearing of the first leg is $360 - 34 = 326°$

ADVANCED MATHEMATICS

21

The answer is option **F**.

Take logs of each side and separate out the left-hand side:

$$x\log_{10} a + 2x\log_{10} b + 3x\log_{10} c = \log_{10} 2$$

$$x(\log_{10} a + 2\log_{10} b + 3\log_{10} c) = \log_{10} 2$$

$x\log_{10}\left(ab^2c^3\right) = \log_{10} 2$, so $x = \dfrac{\log_{10} 2}{\log_{10}\left(ab^2c^3\right)}$

22

The answer is option **D**.

Factorise the numerator and denominator: $\dfrac{(x-2)(x+2)}{x(x-2)}$

Cancel the $(x-2)$ factor to leave $\dfrac{x+2}{x}$

23

The answer is option **D**.

A: $\tan\left(\frac{3\pi}{4}\right) = -1$

B: $\log_{10} 100 = 2$

C: $\sin^{10} x \leq 1$ for any x; in this case, it equals 1

D: $\log_2 10 > \log_2 8 = 3$

E: $0 < \sqrt{2} - 1 < 1$ (as $1 < \sqrt{2} < 2$), so raising it to the power of 10 still gives a number less than 1

So the largest of the five numbers is **D**: $\log_2 10$

24

The answer is option **C**.

Substituting values for x and using the Remainder Theorem gives:

for $x = 2$	$8 + 4p + 2q + p^2 = 0$	(i)
for $x = 1$	$1 + p + q + p^2 = -3.5$	(ii)
Multiply (ii) by –2:	$-2 - 2p - 2q - 2p^2 = 7$	(iii)
Add (i) and (iii):	$6 + 2p - p^2 = 7$	
Rearranging:	$p^2 - 2p + 1 = 7$	
	$(p - 1)^2 = 0$	so $p = 1$

25

The answer is option **E**.

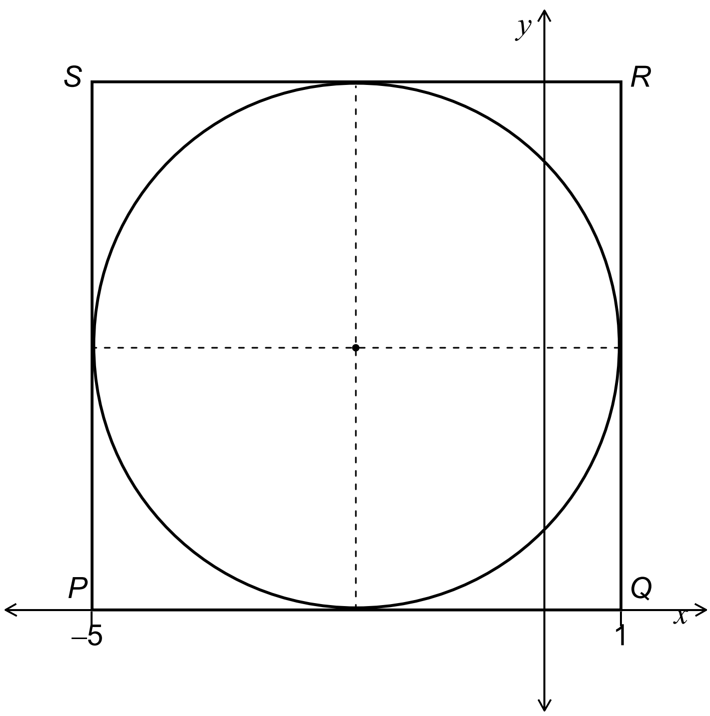
Let $y = 2^x$, then $y^2 - 8y + 15 = 0$. Solve to give $y = 3$ or 5

$2^x = 3$ implies that $x = \dfrac{\log_{10} 3}{\log_{10} 2}$. Similarly, the other solution is $\dfrac{\log_{10} 5}{\log_{10} 2}$

Sum of roots $= \dfrac{\log_{10} 3 + \log_{10} 5}{\log_{10} 2} = \dfrac{\log_{10} 15}{\log_{10} 2}$

26

The answer is option **C**.



The centre of the circle is at (–2, 3)

The side of the square is of length 1 – (–5) = 6

The radius of the circle is $\dfrac{6}{2} = 3$

The equation of the circle is: $(x - (-2))^2 + (y - 3)^2 = 3^2 = 9$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 9$$

$$x^2 + y^2 + 4x - 6y + 4 = 0$$

27

The answer is option **E**.

Statement 1 subtracts $a+b$ from both sides.

Statement 2 can be written as $(a-b)^2 \geq 0$. This is always true.

Statement 3 can be false if c is negative, for example $a = 2$, $b = 1$, $c = -1$

28

The answer is option **C**.

The first term is $a = 8$

The fifth term is $ar^4 = 2$

Combining these two equations gives
$$r^4 = \frac{1}{4}$$
$$r^2 = \frac{1}{2}$$

So $r^2 = \dfrac{1}{\sqrt{2}}$, taking positive square root because sixth term is positive.

Sum to infinity = $\dfrac{a}{1-r} = \dfrac{8}{1-\dfrac{1}{\sqrt{2}}} = 16\left(1+\dfrac{\sqrt{2}}{2}\right) = 8\left(2+\sqrt{2}\right)$

29

The answer is option **A**.

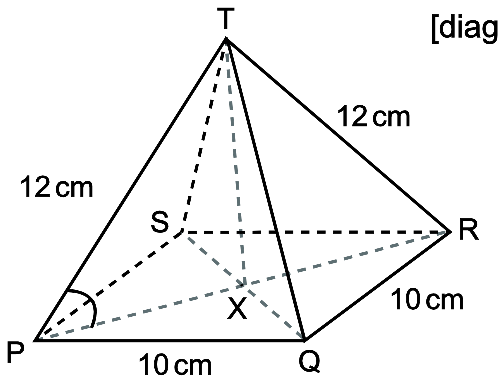
The sequence begins $a_1 = 2$, $a_2 = 2 + (-1) = 1$, $a_3 = 2 = 1 + (-1)^2 = 2$, $a_4 = 1$, etc.

So the sum of the first 100 terms is $2+1+2+1+\ldots = 50\times 2 + 50 \times 1 = 150$

30

The answer is option **F**.

1.

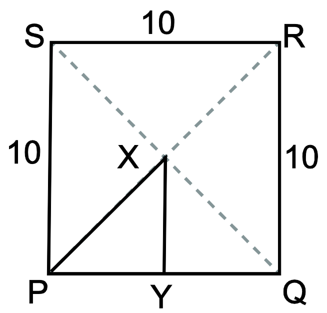


[diagram not to scale]

We need to find angle TPX.

Draw a perpendicular from T to X (midpoint of square PQRS), then find PX.

2.



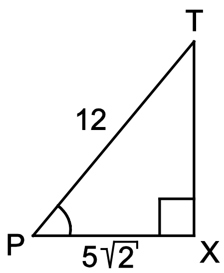
[diagram not to scale]

Using Pythagoras' Theorem

$PX^2 = 5^2 + 5^2 = 50$

so $PX = \sqrt{50} = 5\sqrt{2}$

3.



[diagram not to scale]

Then $\cos TPX = \dfrac{5\sqrt{2}}{12}$ so $TPX = \cos^{-1}\dfrac{5\sqrt{2}}{12}$

31

The answer is option **C**.

To determine how many roots the equation has, we can investigate how many times the graph of $y = x^4 - 4x^3 + 4x^2 - 10$ intersects the x-axis. To help us do this we can find where the graph has turning points:

Differentiating gives: $4x^3 - 12x^2 + 8x = 0$

Solving this: $4x\left(x^2 - 3x + 2\right) = 4x(x-1)(x-2) = 0$

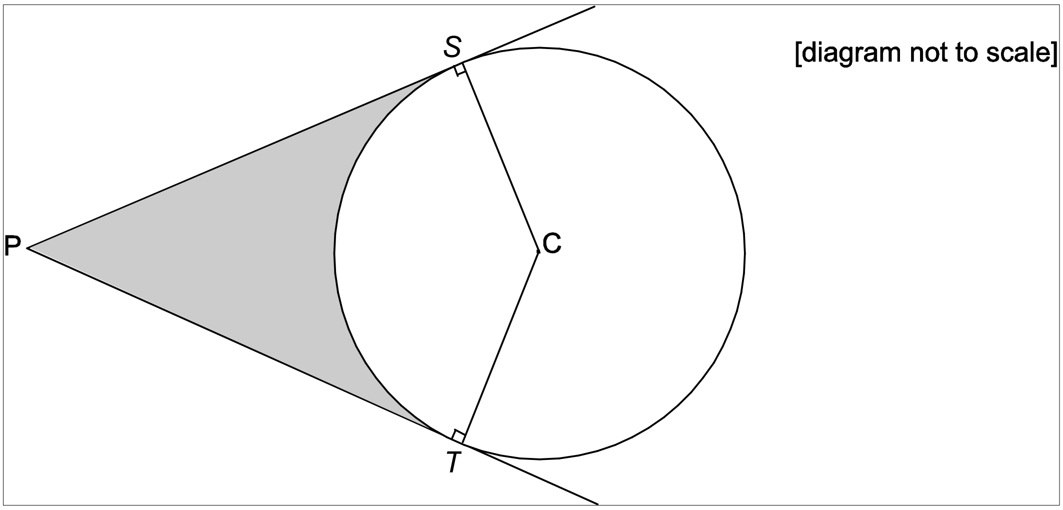
Giving $x = 0$, 1, 2

The coordinates of the turning points are: (0, – 10), (1, – 9), (2, – 10).

From this, and knowing the general shapes of quartics, we can deduce that the curve will intersect the x-axis at two distinct places and so the original quartic must have two distinct real roots.

32

The answer is option **A**.



Let the tangent touch the circle at S and T. The line PT is at right angles to the radius TC.

Using Pythagoras' theorem: $PT^2 + 10^2 = PC^2$

$$PT = \sqrt{400 - 100} = \sqrt{300} = 10\sqrt{3}$$

Angle $PCT = 60°$ because $\cos PCT = \frac{10}{20} = \frac{1}{2}$

The required area = (area of $PTCS$) – (area of the sector SCT)

$$= \left(\frac{1}{2} \times 10\sqrt{3} \times 10 \times 2\right) - \left(\frac{\frac{2\pi}{3}}{2\pi} \times \pi \times 10^2\right)$$

$$= 100\sqrt{3} - \frac{100}{3}\pi = \frac{100}{3}\left(3\sqrt{3} - \pi\right)$$

33

The answer is option **D**.

$$7\cos\theta - 3\tan\theta\sin\theta = 1$$

Using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad 7\cos\theta - 3\dfrac{\sin\theta}{\cos\theta}\sin\theta = 1$

Multiply through by $\cos\theta$ ($\cos\theta \neq 0$) gives: $\quad 7\cos^2\theta - 3\sin\theta\sin\theta = \cos\theta$

$$7\cos^2\theta - 3\sin^2\theta - \cos\theta = 0$$

Using $\sin^2\theta + \cos^2\theta = 1$ $\quad 10\cos^2\theta - \cos\theta - 3 = 0$

Factorising $\quad (5\cos\theta - 3)(2\cos\theta + 1) = 0$

so $\quad \cos\theta = \dfrac{3}{5}$ or $-\dfrac{1}{2}$

34

The answer is option **C**.

If the three (integer) sides are equal there is the 4-4-4 (equilateral) triangle.

If the largest side is 5 cm, then we have two essentially different (non-congruent) triangles possible:

a (right-angled) 3-4-5 triangle (noting that the order of sides is unimportant); and a 2-5-5 (isosceles) triangle.

However, in every other combination of numbers the longest side is exceeded in length by the sum of the two shorter sides. If you imagine constructing the ‘triangle’ by drawing the long side first and then the shorter sides, one at each end of the longest one, then the shorter sides would not meet. So there cannot be a triangle with a side greater than or equal to 6.

35

The answer is option **D**.

For real distinct roots the discriminant condition gives:

$(a-2)^2 > 4a(-2)$ or $(a-2)^2 - 4a(-2) > 0$

$a^2 + 4a + 4 > 0$

$(a+2)^2 > 0$

This is true for all values of a except -2

36

The answer is option **G**.

Rearranging $3x^2 = (a+2)x - 3$ $\quad$ $3x^2 - (a+2)x + 3 = 0$

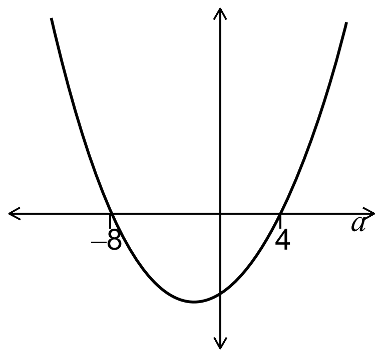
Considering the discriminant $(b^2 - 4ac)$ of the quadratic: $\quad [-(a+2)]^2 - (4 \times 3 \times 3) > 0$

$$a^2 + 4a + 4 - 36 > 0$$

$$a^2 + 4a - 32 > 0$$

$$(a+8)(a-4) > 0$$

Referring to the sketch of the quadratic in a



it is seen that the required set of values is: $a < -8,\ a > 4$

37

The answer is option **B**.

$-1 \le \tan x \le 1$ is satisfied when $0 \le x \le \frac{\pi}{4}$ and $\frac{3\pi}{4} \le x \le \pi$

$\sin 2x \ge 0.5$ is satisfied when $\frac{\pi}{6} \le 2x \le \frac{5\pi}{6}$ so $\frac{\pi}{12} \le x \le \frac{5\pi}{12}$

So both are satisfied together when $\frac{\pi}{12} \le x \le \frac{\pi}{4}$

The length of the interval $= \frac{\pi}{6}$

38

The answer is option **D**.

As the lines are perpendicular, the product of their gradients is –1

So $mp = 1$ $\quad p = -\dfrac{1}{m}$

The lines intersect the x-axis at L and M, when $y = 0$

L: $mx + 3 = 0 \quad x = -\dfrac{3}{m}$

M: $px + 2 = 0 \quad x = -\dfrac{2}{p} = 2m$

As the distance between these points is 5, then $2m + \dfrac{3}{m} = 5$

Rearranging gives: $2m^2 - 5m + 3 = 0$

Factorising to: $(2m-3)(m-1) = 0, m = \dfrac{3}{2}$ or1 $\quad (2m - 3)(m - 1) = 0$

giving $m = \dfrac{3}{2}$ or 1

As m $\neq$ 1, then $m = \dfrac{3}{2}$ giving $p = -\dfrac{2}{3}$

$m + p = \dfrac{3}{2} - \dfrac{2}{3} = \dfrac{5}{6}$

39

The answer is option **E**.

The translation takes the minimum point from (0, 0) to (4, 3).

The reflection in $y = -1$ flips the curve over and the minimum point becomes a maximum at (4, –5).

The curve is now $y = -x^2$ translated by $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ so the equation is: $y + 5 = -(x - 4)^2$

Rearranging to give: $y = -5 - (x - 4)^2$

40

The answer is option **F**.

Since lengths are given to the nearest cm, the minimum possible length of PQ and the maximum possible length of PR are 3.5 and 2.5 respectively.

Use Pythagoras' Theorem: $\text{QR}^2 = 3.5^2 - 2.5^2 = 12.25 - 6.25 = 6$

Take square root: $\text{QR} = \sqrt{6}$